Wall-Following Control of a Two-Wheeled Mobile Robot

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Wall-following control problem for a mobile robot is to move it along a wall at a constant speed and keep a specified distance to the wall. *This paper proposes wall-following controllers based on Lyapunov function candidate for a two-wheeled mobile robot (MR) to follow an unknown wall*. The mobile robot is considered in terms of kinematic model in Cartesian coordinate system. Two wall-following feedback controllers are designed : full state feedback controller and observer-based controller. To design the former controller, the errors of distance and orientation of the mobile robot to the wall are defined, and the feedback controller based on Lyapunov function candidate is designed to guarantee that the errors converge to zero asymptotically. The latter controller is designed based on Busawon's observer as only the distance error is measured. Additionally, the simulation and experimental results are included to illustrate the effectiveness of the proposed controllers.

Key Words: Two-Wheeled Mobile Robot, Wall-Following

Nomenclature

(x, y)	: Fixed coordinates of mobile robot [m]
(X, Y)	: Moving coordinates of mobile robot [m]
r	: Wheel radius [m]
b	: Distance from mobile robot center to driving wheel [m]
W	: A point on reference wall
ρ	: Wall curved radius at point W [m]
$t-t$	: Tangential line with wall at point W
ϕ_w	: Wall orientation at point W [rad]
ϕ	: Head angle of mobile robot [rad]
d	: Distance from mobile robot to $t-t$ [m]
v	: Straight velocity of mobile robot at its center point [m/s]
ω	: Angular velocity of mobile robot at its center point [rad/s]
ω_{rw}, ω_{lw}	: Angular velocities of the right and left wheels [rad/s]
v_w	: Velocity of point W[m/s]
ω_w	: Time rate of the change of $t-t$ direction [rad/s]
e_1, e_2	: Distance and angular errors [m]
v_r	: Constant speed [m/s]
d_r	: Desired distance value [m]
s	: Known function
f_i	: Class C^1 that the first derivatives are continuous with respect to their arguments
$N(s, \xi, t)$	: An observability matrix
S_λ	: A symmetric positive definite matrix
λ	: Positive number
r_s	: Radius of the roller [m]
l_s	: Length of the sensor [m]

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1. Introduction

Wall-following technique is popular and useful for the autonomous navigation in structured known and unknown indoor environment. It can be the case in the following situations (Turennout, Hondred and Schelven, 1992): obstacle avoidance-when the sensors cannot provide an overview of the shape or the size of an obstacle, it becomes necessary to follow the contour of the obstacle until the shape or the size of the obstacle is available; *following an unknown wall*-when there is little or no information about the environment, the trajectory may be specified as following the wall on the right until the first doorway; *following a known wall*-when the trajectory of the robot has been planned and dead-reckoning can be used. The dead-reckoning, however, suffers from accumulating errors, so a way to keep these errors small is by tracking a wall. In addition, there are some wall-following controllers for the mobile robot which have been reported in literature. Turennout, Hondred and Schelven (1992) proposed a scheme that the distance and the orientation of the mobile robot are estimated using a robot model and corrected by the sensor measurement. Medromi, Tigli and Thomas (1994) designed an asymptotic bilinear observer to estimate the position and the orientation of the mobile robot and applied it to the wall-following problem. Bemporad, Marco and Tesi (2000) designed an extended Kalman filter using the combined data of ultrasonic and odometric sensors for the estimation of the robot coordinate. Yata, Kleeman and Yuta (1998) proposed a simple wall-following controller based on an accurate bending angle of the reflecting point with a new ultrasonic sensing method.

This paper proposed nonlinear feedback controllers based on Lyapunov function candidate for a two-wheeled mobile robot to follow an unknown smooth curved wall. The mobile robot is considered in terms of kinematic model in Cartesian coordinate system. The wall-following problem is that the mobile robot moves along a wall at a constant speed and keeps a specified distance to the wall. Two types of feedback controllers were proposed: full state feedback controller and observer-based feedback controller. For the full state feedback controller, the errors of the distance and the orientation of the mobile robot to the wall are defined, and the control law is extracted from the stable condition based on Lyapunov function candidate. In the case of the observer-based controller, Busawon's observer is used for the orientation estimation to derive the control law. Also, a simple way of measuring the errors using two potentiometers is introduced. Additionally, the simulation and the experimental results are given to show the effectiveness of the proposed controllers.

2. Wall-Following Problem

In this section we derive kinematic equation for a two-wheeled mobile robot in Cartesian coordinate system. The mobile robot is modeled under the following assumptions:

(1) The curved radius of the wall is sufficiently larger than turning radius of the mobile robot;
(2) The mobile robot has two driving wheels for its body motion, and those are positioned on the axis passed through the robot geometric center;
(3) Two passive wheels are installed in the front and the rear of the mobile robot for the balance of the mobile platform;
(4) A seam tracking sensor is fixed on the axis of the wheels;
(5) The center of mass and the center of rotation of the mobile robot are coincided;
(6) The velocity at the point contacted with the ground in the plane of the wheel is zero.

The model of a two-wheeled mobile robot following a smooth curved wall in the Castesian coordinate system is described in Fig. 1.

The ordinary form of a mobile robot with two driving wheels can be derived as follows (Fukao, Nakagawa and Adachi, 2000):

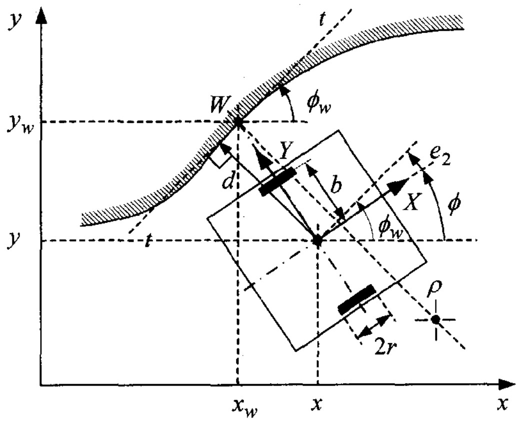

Fig. 1 The model of the MR following the wall

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 \\ \sin\phi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \qquad (1)$$

The relationship between v, ω and the angular velocities of two driving wheels can be given by :

$$\begin{bmatrix} \omega_{rw} \\ \omega_{lw} \end{bmatrix} = \begin{bmatrix} 1/r & b/r \\ 1/r & -b/r \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \qquad (2)$$

A point $W(x_w, y_w)$ is changed according to the motion of the mobile robot with the following dynamics :

$$\begin{cases} \dot{x}_w = v_w \cos\phi_w \\ \dot{y}_w = v_w \sin\phi_w \\ \dot{\phi}_w = \omega_w \end{cases} \qquad (3)$$

The equation of the tangential line $t-t$ with the wall at point W is as follows :

$$(x-x_w)\sin\phi_w - (y-y_w)\cos\phi_w = 0 \qquad (4)$$

The distance from the mobile robot to this line, or to the wall, is the following form :

$$d = (x-x_w)\sin\phi_w - (y-y_w)\cos\phi_w \qquad (5)$$

Wall-Following Problem : Given a smooth curved infinite wall described in Cartesian plane as shown in Fig. 1, determine a feedback control law such that the mobile robot with Eq. (1) moves at a constant speed v_r along the wall and keeps the distance Eq. (5) to be a desired value d_r.

To solve the wall-following problem, let the errors be

$$\begin{cases} e_1 = d - d_r \\ e_2 = \phi - \phi_w \end{cases} \qquad (6)$$

and the problem becomes to design a controller which obtains $e_1 \to 0$ and $e_2 \to 0$ as $t \to \infty$.

3. Controller Design

In this section, we introduce two controllers which solve the wall following problem : full state feedback controller and observer-based feedback controller.

3.1 Full state feedback controller

From Eqs. (1), (3) and (5), we have

$$\begin{aligned} \dot{d} &= (\dot{x}-\dot{x}_w)\sin\phi_w + (x-x_w)\cos\phi_w\omega_w \\ &\quad -(\dot{y}-\dot{y}_w)\cos\phi_w + (y-y_w)\sin\phi_w\omega_w \\ &= -v\sin(\phi-\phi_w) + (x-x_w)\cos\phi_w\omega_w \\ &\quad + (y-y_w)\sin\phi_w\omega_w \\ &= -v\sin(\phi-\phi_w) + d\omega_w\tan(\phi-\phi_w) \end{aligned}$$

where

$$x - x_w = d\frac{\cos(\pi/2-\phi)}{\cos(\phi_w-\phi)} = d\frac{\sin\phi}{\cos(\phi_w-\phi)}$$

and

$$y - y_w = -d\frac{\sin(\pi/2-\phi)}{\cos(\phi_w-\phi)} = -d\frac{\cos\phi}{\cos(\phi_w-\phi)}$$

Hence the error dynamics can be represented as follows :

$$\begin{cases} \dot{e}_1 = -v\sin e_2 + (e_1+d_r)\omega_w\tan e_2 \\ \dot{e}_2 = \omega - \omega_w \end{cases} \qquad (7)$$

To design the full state feedback controller, the Lyapunov function candidate is chosen as

$$V = \frac{1}{2}e_1^2 + \frac{1-\cos e_2}{k_2} \geq 0 \qquad (8)$$

The derivative of V yields

$$\dot{V} = \frac{1}{k_2}\sin e_2\left[-ve_1k_2 + (e_1+d_r)\omega_w\frac{1}{\cos e_2}e_1k_2 + (\omega-\omega_w)\right] \qquad (9)$$

To obtain $\dot{V} \leq 0$, we choose the control law as follows

$$\begin{cases} v = v_r \\ \omega = k_2[v_r - (e_1+d_r)\omega_w/\cos e_2]e_1 - k_1\sin e_2 + \omega_w \end{cases} \qquad (10)$$

where k_1 and k_2 are positive values.

3.2 Observer-based feedback controller

In this section the state estimation of a nonlinear system is discussed. The term of full state feedback is used to denote a system in which all states are available for feedback. In practice, the full state measurements are not always possible, so it is needed to estimate the state that cannot be measured.

It is assumed that the curved wall is smooth. Without loss of practical aspect, it can be assumed that when the system operates around steady state, the angular error e_2 between the heading angle of the mobile robot and the tangential angle of the wall at W is small. Then using $\sin e_2 \approx e_2$ and $\tan e_2 \approx e_2$, we can rearrange Eqs. (7) and (10) as follows:

$$\begin{cases} \dot{e}_1 = [-v + (e_1 + d_r)\omega_w] e_2 \\ \dot{e}_2 = \omega - \omega_w \end{cases} \tag{11}$$

The Lyapunov function candidate can be chosen as

$$V = \frac{1}{2} k_2 e_1^2 + \frac{1}{2} e_2^2 \tag{12}$$

and the following controller can be derived as follows:

$$\begin{cases} v = v_r \\ \omega = k_2 [v_r - (e_1 + d_r)\omega_w] e_1 - k_1 e_2 + \omega_w \end{cases} \tag{13}$$

For general nonlinear system, a high gain observer is used to handle part of the nonlinearity of the system by choosing a sufficiently large value of a given design parameter. The following briefly summarize the high gain observer for a class of nonlinear systems in special canonical observable form which studied by Busawon, Farza and Hammouri (1997): Consider a single-output system

$$\dot{z} = F(s, z) z + G(u, s, z) \tag{14}$$

$$y = Cz \tag{15}$$

where

$$F(s, z) = \begin{bmatrix} 0 & f_1(s, z_1) & 0 & \cdots & 0 \\ 0 & 0 & f_2(s, z_1, z_2) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & f_{n-1}(s, z) \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \tag{16}$$

$$G(u, s, z) = \begin{bmatrix} g_1(u, s, z_1) \\ \vdots \\ g_{n-1}(u, s, z_1, z_2, \cdots, z_{n-1}) \\ g_n(u, s, z_1, z_2, \cdots, z_n) \end{bmatrix} \tag{17}$$

$$\begin{aligned} &C = [1 \ \ 0 \ \ \cdots \ \ 0] \\ &z = (z_1, z_2, \cdots, z_n) \in R^n,\ u \in R^m,\ y \in R \end{aligned} \tag{18}$$

s is a known function and f_i is a class C^1 that the first derivatives are continuous with respect to their arguments.

Let $N(s, \xi, t)$ be observability matrix for the system of Eqs. (14) and (15):

$$\begin{aligned} &N(s, \xi, t) = diag[1, f_1(s, \xi), f_1(s, \xi) \\ &f_2(s, \xi), \cdots, f_1(s, \xi) f_2(s, \xi) \cdots f_{n-1}(s, \xi)] \end{aligned} \tag{19}$$

Consider the following algebraic Lyapunov equation

$$\frac{dS_\lambda}{dt} = \lambda S_\lambda + A^T S_\lambda + S_\lambda A - C^T C = 0 \tag{20}$$

where S_λ is the symmetric positive definite matrix, λ is a positive number and matrix A is of the form:

$$A = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix} \tag{21}$$

We assume the following conditions:

(A1) There exists a subset $U \subset L^\infty(R^+, R^m)$ and two compact sets $K_1 \subset K_2$ such that every trajectory $z(t)$ associated to any $u \in U$ and issued from K_1 to K_2.

(A2) $s(t)$ is known and bounded in a class C^1, $\dot{s}(t)$ is bounded.

(A3) $\exists \alpha_1 > 0, \forall u \in R^m, \forall z \in R^n, \forall t \geq 0 : |f_i(s(t), z)| > \alpha_1$

(A4) $\exists \alpha_2 > 0, \forall u \in R^m, \forall z \in R^n, \forall t \geq 0 : |g_i(u, s(t), z)| > \alpha_2$ for i=1, ⋯, n.

(A5) $\exists \beta, \gamma \geq 0; \forall u \in R^m, \forall z \in R^n; \forall t \geq 0$

$$\left\| \frac{\partial f_j(s(t), z)}{\partial z} \right\| \leq \beta, \left\| \frac{\partial g_i(u(t), s(t), z)}{\partial z} \right\| \leq \gamma$$

for $j = 1, \cdots, n-1$ and i=1, ⋯, n.

(A6) $N(s, \xi, t)$ is full rank for all $t \geq 0$ and its time derivative is bounded.

Busawon's observer is of reduced observer designed for a class of nonlinear system; however, its mechanism estimates all state, like a full order observer. Hence, the wall-following observer is also solved by the estimation of all errors, that is, $\hat{e}_1$ and $\hat{e}_2$. The real error e_1 and its estimation $\hat{e}_1$ are different during the transient time only but same during the steady time, and they, in turn, are used to estimate the other error, $\hat{e}_2$. The stability of the system based on Busawon observer was proved in the work of Busawon, Farza, and Hammouri (1997).

It is assumed that the system Eq. (13) satisfies the assumptions. Then, there exists $\lambda>0$ such that the following equation is an exponential observer for the system:

$$\dot{\hat{z}}=F(s,\hat{z})\hat{z}+G(u,s,\hat{z})-N^{-1}(s,\hat{z},t)S_\lambda^{-1}C^T(C\hat{z}-y) \quad (22)$$

with

$$\hat{z}=\hat{e}=\begin{bmatrix}\hat{e}_1\\ \hat{e}_2\end{bmatrix} \quad (23)$$

$$F(\hat{z})=\begin{bmatrix}0 & \hat{f}_1\\ 0 & 0\end{bmatrix},\ \text{where } \hat{f}_1=(\hat{e}_1+d_0)\,\omega_w-v \quad (24)$$

$$G(\hat{z})=\begin{bmatrix}0\\ \omega-\omega_w\end{bmatrix},\ C=[1\ \ 0],\ N^{-1}(\hat{z})=\begin{bmatrix}1 & 0\\ 0 & \frac{1}{\hat{f}_1}\end{bmatrix} \quad (25)$$

$$S_\lambda^{-1}=\begin{bmatrix}2\theta & \theta^2\\ \theta^2 & \theta^3\end{bmatrix},\ \text{where } \theta>0 \text{ is an observer gain.} \quad (26)$$

The observer Eq. (22) becomes

$$\begin{cases}\dot{\hat{e}}_1=[-v+(\hat{e}_1+d_r)\,\omega_w]\,\hat{e}_2-2\theta(\hat{e}_1-e_1)\\ \dot{\hat{e}}_2=-\dfrac{\theta^2(\hat{e}_1-e_1)}{-v+(\hat{e}_1+d_r)\,\omega_w}-\omega_w+\omega\end{cases} \quad (27)$$

and the observer-based feedback controller can be chosen as

$$\begin{cases}v=v_r\\ \omega=k_2[v_r-(\hat{e}_1+dr)\,\omega_w]\,\hat{e}_1-k_1\hat{e}_2+\omega_w\end{cases} \quad (28)$$

where k_1 and k_2 are positive values.

4. Simulation and Experimental Results

4.1 Measurement of the errors

To derive the controller, the tracking errors must be measured; that is to say, position of the mobile robot relative to the reference path must be known. In this paper, a simple error measurement scheme is proposed as shown in Fig. 2, and the sensor is designed using potentiometers to get the errors e_1 and e_2 for the controllers Eqs. (13) and (28) as shown in Fig. 3. Two potentiometers are used for measuring the errors: one

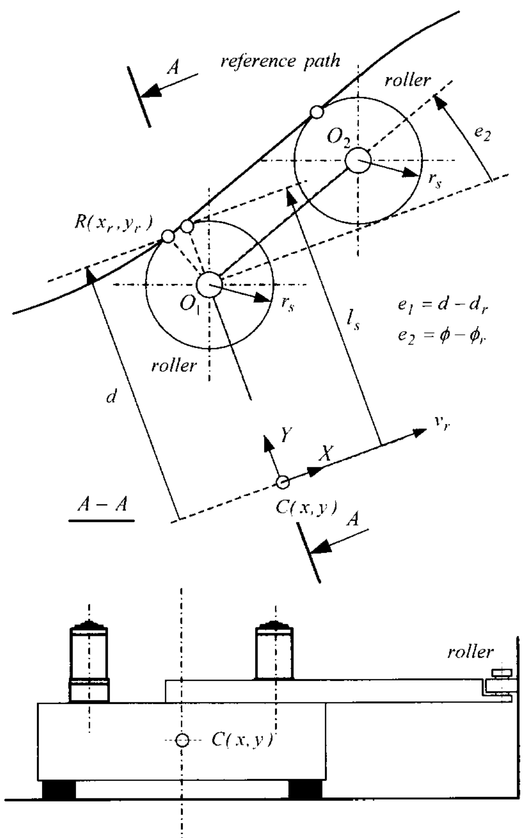

Fig. 2 Scheme for measuring the errors

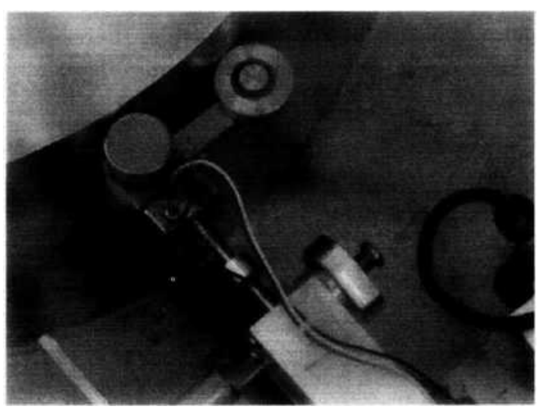

Fig. 3 The sensor for measuring the errors

linear potentiometer for measuring distance d and one angular potentiometer for measuring the angular error, e_2, of the mobile robot to the wall. The potentiometers are setup to touch the wall by two rollers at the points O_1 and O_2; the distance between them, O_1O_2, is chosen according to the curved radius of the wall at the contact point R (x_r, y_r). The roller's diameter is chosen small enough to overcome the friction force.

We have the relationship

$$\begin{cases} e_1 = d - d_r = [l_s - r_s(1 - \cos e_2)] - d_r \\ e_2 = \angle(O_1C, O_1O_2) - \pi/2 \end{cases} \tag{29}$$

where r_s is the radius of the roller, and l_s is the length of sensor.

4.2 Configuration of the control system

For the control system, a PIC-based controller is developed. The controller is composed of two parts: servo controller and main controller. The block diagram of the total control system is shown in Fig. 4. The servo controller integrates two PIC16F877 microcontrollers for the motor control of the left and right wheels. The motors are driven via LMD18200 Dual Full-Bridge driver. The servo controller can perform indirect servo control using one encoder. Also, the main controller using another PIC16F877 links to the servo controllers via I2C communication. In addition, two A/D ports on main controller are connected to two potentiometers for sensing the errors as mentioned in sec. 4.1.

To verify the effectiveness of the proposed controllers, the simulation and experimental results are done for a mobile robot following an unknown wall with full state feedback controller

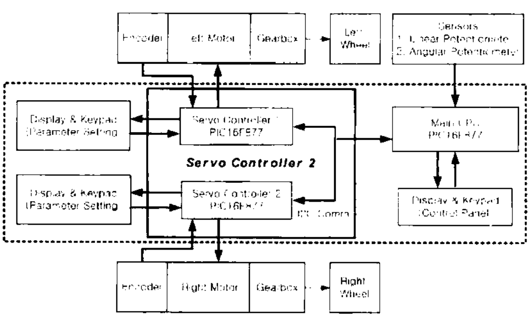

Fig. 4 Block diagram of the control system

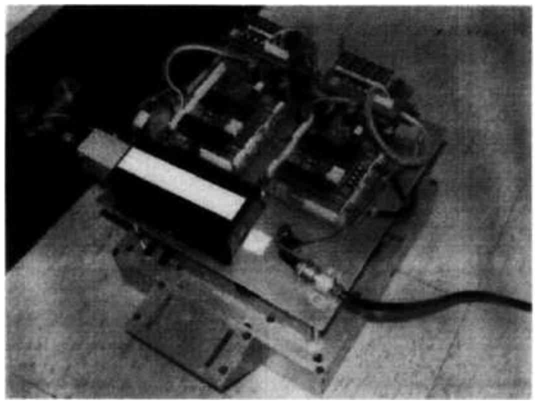

Fig. 5 The experimental mobile robot following the straight wall

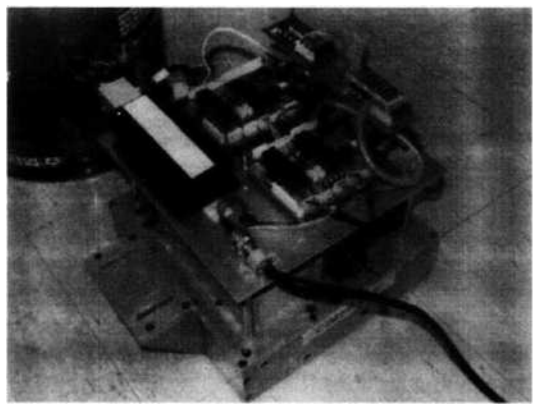

Fig. 6 The experimental mobile robot following the round wall.

and observer-based feedback controller. The parameters of the mobile robot are $b = 105mm$ and $r = 25mm$. The desired values of the velocity and the distance are $v_r = 75mm/s$ and $d_r = 240$ mm. The experimental mobile robot in the case of following a straight wall and a round wall are shown in Figs. 5 and 6, respectively.

4.3 Full state feedback controller

In this section, the controller Eq. (10) is used for the mobile robot to follow a straight wall with $\phi_w(0) = 30°$ and a round wall with radius of $150mm$. The positive constants in the controller are chosen as $k_1 = 1.25$ and $k_2 = 250$. The initial values are $d(0) = 230mm$ and $\phi(0) = 15°$. The simulation and experimental results for the st-

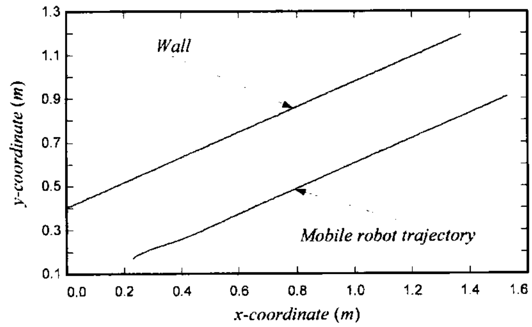

Fig. 7 The trajectory of the MR (straight wall)

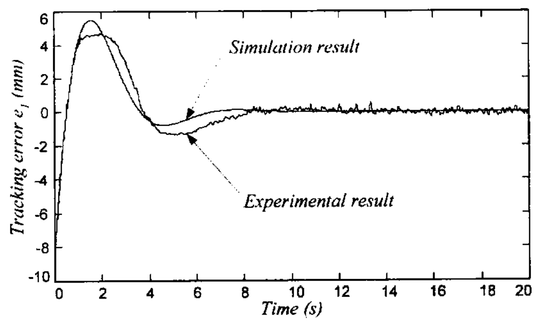

Fig. 8 The tracking error e_1 (straight wall)

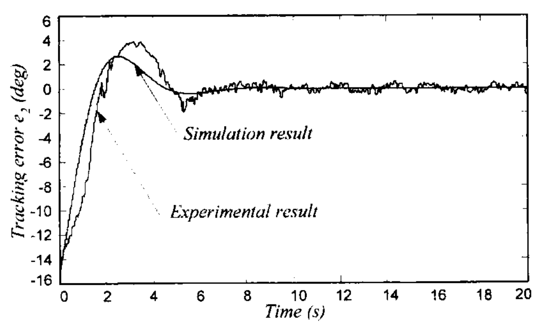

Fig. 9 The tracking error e_2 (straight wall)

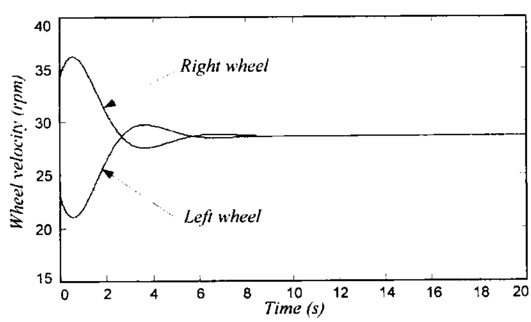

Fig. 10 Wheels' velocities of MR (straight wall)

raight wall are shown in Figs. 7~10, and, for the round wall, in Figs. 11~13. For the straight wall, the trajectory of the mobile robot with respect to the wall are given in Fig. 7 ; the simulation and experimental results of the errors are given in Figs. 8 and 9 ; and the control inputs, the velocities of the left and right wheels, are given in Fig. 10.

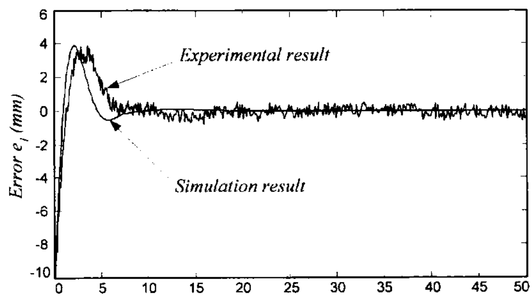

Fig. 11 Tracking error e_1 (round wall)

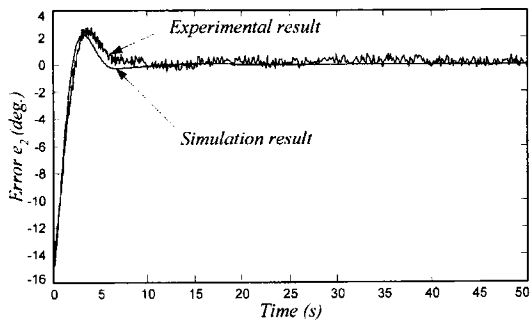

Fig. 12 Tracking error e_2 (round wall)

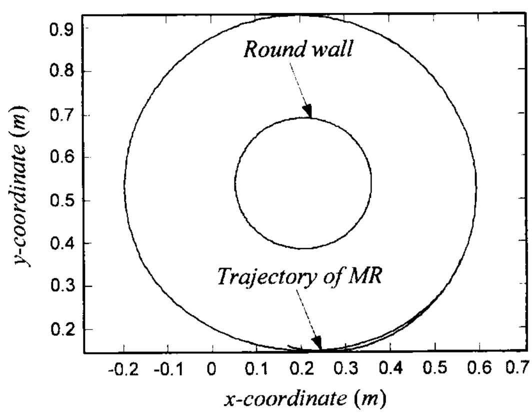

Fig. 13 Trajectory of the MR (round wall)

The experiment is also done for the mobile robot to follow a round wall with the radius of 150mm for 50 seconds. The initial values are $d(0)=230mm$ and $\phi(0)=15°$. The tracking errors are shown in Figs. 11 and 12 ; the trajectory of the mobile robot is given in Fig. 13. And it can be seen that the errors converge to zero after about 6 seconds.

4.4 Observer-based feedback controller

In this section, the controller Eq. (28) is used for which the estimated error dynamics, $\hat{e}_1$ and $\hat{e}_2$, are calculated as Eq. (27). The simulation and experimental results for observer-based controller are shown in Figs. 14~17. The positive constants in the controller are chosen as $k_1=12.5$ and $k_2=2500$. The initial values are $d(0)=230mm$ and $\phi(0)=5°$. The trajectory of the mobile robot following the straight wall is shown in Fig. 14 ; the simulation and experimental results of errors are shown in Figs. 15 and 16 ; and Fig. 17 shows the control input.

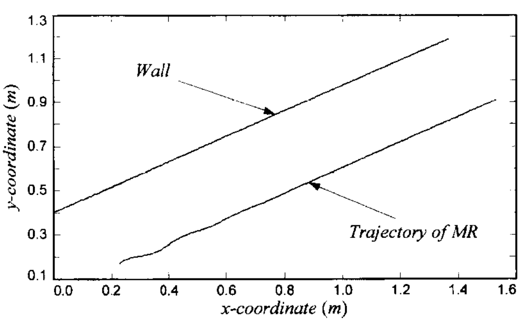

Fig. 14 The trajectory of the MR for observer-based controller

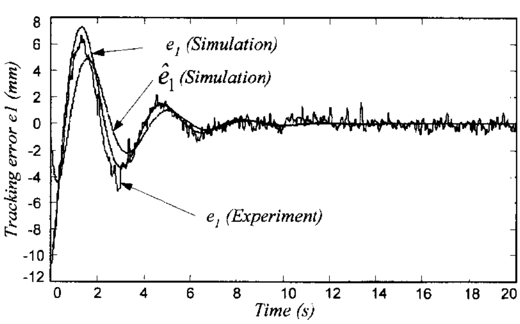

Fig. 15 Tracking errors e_1 for observer-based controller

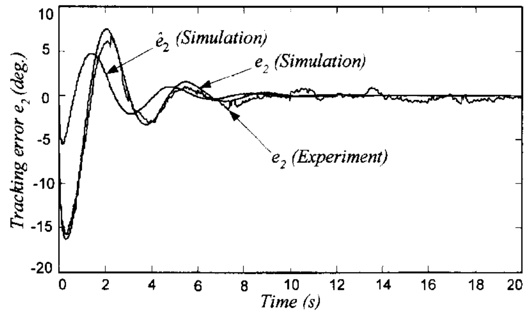

Fig. 16 Tracking errors e_2 for observer-based controller

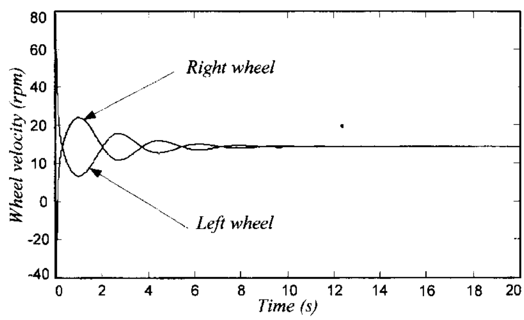

Fig. 17 The wheels' velocities of the MR for observer-based controller

From the simulation and experimental results we can conclude that

- The full state feedback controller can be used for a mobile robot to follow any smooth curved wall.
- The observer-based feedback controller can be used when only information from the mobile robot to the wall is available with the assumption that the initial angular error, $e_2(0)$, is small enough.

5. Conclusions

This paper introduced two wall-following controllers for the two-wheeled mobile robot. The mobile robot can move along a wall at a constant speed and keep a specified distance to the wall. To design the full state feedback controller, the distance and the orientation of the mobile robot are needed. In observer-based controller, only the distance information is measured, and Busawon's observer is introduced to estimate the orientation of the mobile robot. The controlled system is stable in the sense of Lyapunov stability. The controllers are verified by means of simulation and experimental results. And it can be seen that the derived model and the proposed controllers can be implemented in a practical mobile robot.

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